THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 3030 Abstract Algebra 2024-25
Tutorial 5
10th October 2024

- Tutorial exercise would be uploaded to blackboard on Mondays provided that there is a tutorial class on that Thursday. You are not required to hand in the solution, but you are advised to try the problems before tutorial classes.
- Please send an email to echlam@math.cuhk.edu.hk if you have any questions.

1. Suppose that $N \triangleleft G$ and $N \cap G' = \{e\}$ where G' is the commutator subgroup of G, show that $N \leq Z(G)$.
2. (a) Let H, K be normal subgroups of G, define $\phi : G/(H \cap K) \to G/H \times G/K$ by $\phi(aH \cap K) = (aH, aK)$. Prove that ϕ is a well-defined and injective homomorphism.
   (b) Prove that ϕ is surjective if and only if $G = HK$.
   (c) Prove that $\mathbb{Z}_{pq} \cong \mathbb{Z}_p \times \mathbb{Z}_q$ for distinct prime numbers p, q.
3. Prove that the group of upper triangular matrices $B_2 \leq GL(2, \mathbb{C})$ is solvable.
4. Suppose G is a finite solvable group, let $0 \neq N \triangleleft G$ is a minimal normal subgroup, i.e. there is no proper nontrivial subgroup $M \leq N$ so that M is normal in G, prove that N is abelian.
5. Does $\mathbb{Q}$ have a composition series?
6. Find a composition series for D_8 the symmetry group of regular 8-gons, and also a composition series for $\mathbb{Z}_{48}$.
7. Let $f : G \to H$ be a homomorphism, if G is solvable, show that the image $f(G)$ is also solvable.